MATH 5C - FINAL EXAM v1 Summer 2023

200 POINTS

NAME: ____________________

Instructions on Canvas. SHOW **ALL** WORK. Each problem worth 20 points.

(1)

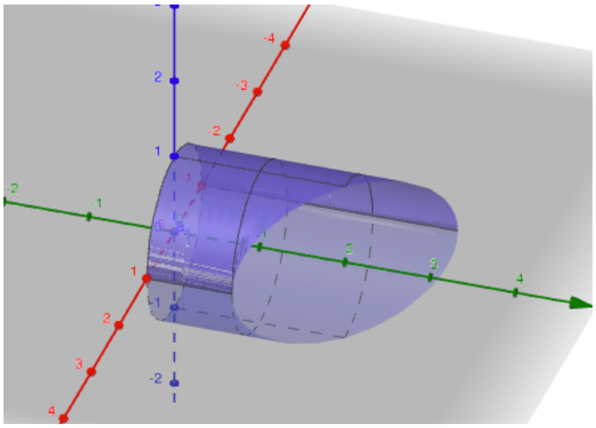
(a) Find the equation of the plane containing the lines.

$$L_1 \begin{cases} x = 2 + t \\ y = 3 - 2t \\ z = 1 + t \end{cases} \qquad L_2 \begin{cases} x = 3 + 4s \\ y = -4 - 8s \\ z = 2 + 4s \end{cases}$$

(b) Find an equation for the tangent plane to the surface $x = y^2 + z^2 + 1$ at the point $(3,1,-1)$

(2) Given: $\vec{F}(x,y,z)=\langle x,\ z,\ 0\rangle$, and surface S which is portion of the cylinder $x^2+z^2=1$, bounded by the planes $y=0$ and the plane $x+y=2$ oriented outward,as shown. Note: this surface in not closed. It is the cylindrical sides only.

Evaluate the flux, $\iint_S \vec{F}\bullet d\vec{S}$



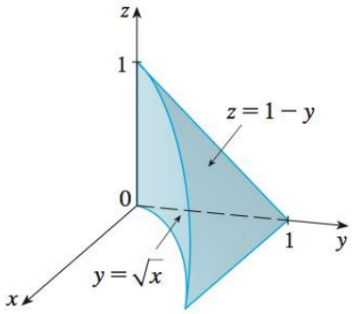
(3) The position vector of a particle is $\vec{r}(t) = \left\langle t^2, \ln t, 2t \right\rangle$

(a) Find the length of the curve for $1 \le t \le 2$

(b) Find parametric equations of the line tangent to $\vec{r}(t)$ at t=1.

(4) Find the maximum volume of a rectangular box that can be inscribed in a sphere of radius 3. Show how you know it is an absolute maximum.

(5) Let E be the solid shown.



a) Set up only: $\iiint_E z\,dV$ Triple integral- rectangular coordinates; order dz dy dx

b) Set up only: $\iiint_E z\,dV$ Triple integral- rectangular coordinates; order dy dz dx

c) Compute $\iiint_E z\,dV$ using any order you wish. (Note, some orders are messier than others).

(6) A hiker at the point (1,2,1) on the hill $z = 6x - x^2 - y^2$ (where the z axis points up, the y axis north, the x axis east)

(a) Find $\left.\frac{\partial z}{\partial x}\right|_{(1,2)}$. Explain what this represents physically.

(b) If the hiker heads north from the point (1,2,1), will she be going up the hill or down? at what rate?

(c) If the hiker heads in the direction from (1,2) towards (5,5) is she going up the hill or down? at what rate?

(d) What is the direction of steepest climb?

(7) Given $f(x,y) = e^x - xe^y$

a) Find all local extrema and saddle points.

b) Compute $\int_0^1 \int_0^4 f(x,y)\, dy\, dx$

(8 and 9) Given the vector field $\vec{F}(x,y) = \langle 6x + y,\, x - 2y \rangle$ and the curve C given by

$\vec{r} = \langle 2\cos t, 2\sin t \rangle \quad 0 \le t \le \pi$

Compute the work $\int_C \vec{F} \bullet d\vec{r}$ two different ways. Be sure to explain clearly what method you are using. (Not just two different parameterizations for the same curve)

(8)

(9)

(10) Find the volume of the solid bounded by the paraboloids $z = 2x^2 + 2y^2$ and. $z = 6 - x^2 - y^2$